Inclusive and Exclusive Semi-Leptonic Decays of B Mesons

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Abstract

In this paper, the semileptonic decays of heavy mesons are treated fully relativistically. By means of an effective vertex, the effect of Fermi momentum are included both at the inclusive and at the exclusive levels, and the spin of both parent and daughter particles are taken into account. The differential decay rates with respect to the lepton energy and momentum transfer are compared with data from ARGUS and CLEO.

Introduction

There are several reasons why semileptonic B decays are of interest. For one thing, the are a small variety of decay products, namely those which contain charm quarks (D, D^*, D^{**}, etc.) and those which do not (π's etc.). V_{ub}/V_{cb} can be determined from the relative number of these decays. In addition, the heavy masses of the b and c quarks suggest that one might be able to apply perturbative QCD to calculate the strong corrections to these processes. This hope has been recently formalised in Heavy Quark Effective Theory[2]. Finally, there's the fact that theoretical uncertainties are much smaller than in non-leptonic decays which contain a wider variety of hadronic decay products.

If we write out a parameterisation for the CKM Matrix, we see that it depends on a complex phase which is resposible for CP violation in the standard model. The magnitudes of the values for the CKM matrix elements place limits on the size of this phase and, thus, on the amount of CP violation in the standard model. According to the particle data group, $V_{ub} = .0035 \pm .0015$ and $V_{cb} = .040 \pm .08$. Recent values of V_{ub} and V_{cb} in the literature fall in this range [3-8].

In studying semi-leptonic decays, the first approximation is to neglect QCD and use a spectator model in which the up quark of the B meson is not involved in the decay except to recombine with the charm quark. In such a model, the decay of the B meson into a D meson reduces to that of the decay of a bottom quark into a charm quark. Quantities that can be determined directly from experimental data include the square of the momentum transfer,

$$\begin{aligned} Q^2 &= (B-D)^2 \\ &= m_B^2 + m_D^2 - 2E_B E_D + 2\vec{p}_B \cdot \vec{p}_D, \end{aligned} \tag{1}$$

and the lepton energy, E_l.

Inclusive Case

Now that we have a process involving quanties that can be determined from experiment, we would want to come up with a theory that relates these quanties. One such model was the one devised by Altarelli, Cabbibo, Corbo, Maiani and Martinelli in 1982[9]. In this model, the bottom quark was assumed to be on shell and, thus, given by

$$\begin{aligned} m_b^2 &= (B-u)^2 \\ &= m_B^2 + m_u^2 - 2m_B\sqrt{p_u^2 + m_u^2} \end{aligned} \tag{2}$$

in the B rest frame. This assumption has the advantage that it avoids having the decay rate depend on an arbitrary overall $1/m_b^5$ that appears in a purely partonic treatment of these

decays[10] The up quark momentum was then assumed to obey a Gaussian distribution,

$$\phi(p) = \frac{1}{\pi^{\frac{3}{2}} p_f^3} exp\left(\frac{-p^2}{p_f^2}\right) \tag{3}$$

which is normalised according to

$$\int d^3p\phi(p) = 1 \tag{4}$$

and is thus fixed up to an adjustable parameter, p_f, known as the Fermi momentum.

In our case, we want to consider the up quark as being more than a mere spectator: instead we write the effective $\overline{u}Bb$ vertex as $\gamma_5 V_B(p_u)$. The decay rate in this model is

$$\Gamma(B \to \overline{u}cl\overline{\nu}) = \frac{N|V_{cb}|^2|V_B|^2}{2m_B(2\pi)^8} \int \frac{d^3p_c}{2E_c}\frac{d^3p_l}{2E_l}\frac{d^3p_\nu}{2E_\nu}\frac{d^3p_u}{2E_u}\phi(p_u)|M|^2\delta^4(B-u-c-l-\nu) \tag{5}$$

where

$$|M|^2 = G_F^2 L_{\alpha\beta}H^{\alpha\beta} \tag{6}$$

and

$$H^{\alpha\beta} = Tr[\gamma^\alpha(1-\gamma_5)(\not{c}+m_c)\gamma^\beta(1-\gamma_5)(\not{b}+m_b)\gamma_5(-\not{u}+m_u)\gamma_5(\not{b}+m_b)]. \tag{7}$$

It turns out this this integral is difficult to evaluate, the problem being that the up and charm quarks are not being assumed to combine into a specific meson. Instead, the quarks are combining to form a cluster X with four momentum $X = u+c$ and mass $m_X^2 = (u+c)^2$. This m_X is arbitrary save for the fact that, experimentally, $m_X > m_D = 1.8963$ GeV while energy conservation requires that $m_X < m_B$. As a result, we will want to rewrite the hadronic phase space so that m_X is integrated over this range. Using standard cluster decomposition techniques, the decay rate becomes:

$$\begin{aligned}\Gamma(B \to Xl\overline{\nu}) = \frac{N|V_{cb}|^2|V_B|^2}{2m_B} \int \quad & |M|^2 \quad \phi(p_u)\frac{1}{(2\pi)^2}d^4p_c d^4p_u \delta^4(X-u-c)\delta(c^2-m_c^2)\delta(u^2-m_u^2) \\ & \times \quad \frac{1}{(2\pi)^2}d^4p_l d^4p_\nu\delta^4(Q-l-\nu)\delta(l^2)\delta(\nu^2) \\ & \times \quad \frac{1}{(2\pi)^2}d^4Qd^4X\delta^4(B-Q-X)\delta(Q\cdot Q-Q^2)\delta(X^2-m_X^2) \\ & \times \quad \frac{1}{(2\pi)^2}dQ^2dm_X^2\end{aligned}$$

Using[10]

$$\int d_2(X \to ab) \quad = \quad (\pi/2)\lambda^{\frac{1}{2}}(1, a^2/X^2, b^2/X^2)\frac{d\Omega}{2\pi}, \tag{8}$$

we get

$$\frac{1}{(2\pi)^2}d^4Qd^4X\delta^4(B-Q-X)\delta(Q\cdot Q-Q^2)\delta(X^2-m_X^2)=\frac{1}{2\pi}\frac{p_Q}{2m_B} \tag{9}$$

where $p_Q = \lambda^{\frac{1}{2}}(1, X^2/m_B^2, Q^2/m_B^2)m_B = p_X$. The remaining delta functions are

$$\begin{aligned}\delta(c^2-m_c^2) &= \delta((X-u)^2-m_c^2)\\ &= \delta(X^2+m_u^2-2E_XE_u+2p_Xp_ucos\theta_{Xu}-m_c^2)\end{aligned}$$

and

$$\begin{aligned}\delta(\nu^2) &= \delta((Q-l)^2)\\ &= \delta(Q^2-2E_QE_l+2p_QE_lcos\theta_{Ql})\end{aligned}$$

These cancel with the cosine integrations in d^3p_u and d^3p_l. The final expression for the decay rate is, thus,

$$\Gamma(B\to Xl\overline{\nu})=\frac{N|V_{cb}|^2V_B^2}{(2\pi)^6(2m_B)^2}\int|M|^2\phi(p_u)\frac{p_udp_udE_ld\phi}{16p_QE_u}dQ^2dm_X^2 \tag{10}$$

If we now compare this formula with data from ARGUS[11][12] and CLEO[13] then we find, after minimising with respect to the parameters m_u, m_c, m_b, p_f and $|V_{ub}|/|V_{cb}|$, that we get a good fit for parameters in the ranges

$$\begin{aligned}m_u &= .13 \pm .38 \text{ GeV}\\ m_c &= 1.4 \pm .4 \text{ GeV}\\ m_b &= 4.9 \pm .3 \text{ GeV}\\ p_f &= .5 \pm .1 \text{ GeV}\\ \text{and } V_{ub}/V_{cb} &= .07 \pm .05\end{aligned}$$

Note that the ARGUS and CLEO data include contributions from $b \to cl\overline{\nu}$ and $b \to ul\overline{\nu}$ decays. In each case, the measured electrons were separated into different categories

including electrons from non-$\Upsilon(4S)$ events, ψ or $\psi(2S)$ decay, τ decay or semileptonic D_s decay, semileptonic D decay, $\pi^0 \to e^+e^-$ decay and semileptonic B decay, the latter being the ones that are used to make these plots. Additional background comes from having hadrons misidentified as electrons. Note that for $E_l > 2.4$ GeV, electrons from B decay can only come from charmless semi-leptonic decays.

Using these parameters and taking the areas under the curves gives us the branching ratio $Br(B \to c\overline{u}l\overline{\nu})$=10.09% and $Br(B \to u\overline{u}l\overline{\nu})$ =.16%. Using

$$\Gamma(B \to \overline{u}cl\overline{\nu}) = Br(B \to \overline{u}cl\overline{\nu})/\Gamma_B \tag{11}$$

and knowing[1] that $\Gamma_B = (1.52 \pm .11) \times 10^{-12}(1.52 \times 10^{24})$ GeV, $|V_{cb}|$ can be calculated to be $.034 \pm .003$.

Exclusive Case

In the spectator model, one can differentiate between D and D^* mesons according to whether the daughter meson has spin 0 or 1. That one can do this was overlooked in a recent paper by V. Barger *et al* that attempted to differentiate between different decay products in the differential m_X distribution[14]. Mahiko Suzuki[15] used this observation to calculate exclusive rates at zero Fermi momentum. In this frame,

$$H^{\alpha\beta} = M_0^\alpha M_0^\beta + M_1^\alpha M_1^\beta \tag{12}$$

where

$$M_0^\lambda \quad \propto \quad Tr\left[(\not{c} + m)\gamma^\lambda(1-\gamma_5)(\not{b} + M)\right]/[4M\{2m(E_c + m)\}^{\frac{1}{2}}] \tag{13}$$

and

$$M_1^\lambda \quad \propto \quad Tr\left[(\not{c} + m)\gamma_5\not{\epsilon}\gamma^\lambda(1-\gamma_5)(\not{b} + M)\right]/[4M\{2m(E_c + m)\}^{\frac{1}{2}}]. \tag{14}$$

Here ϵ_λ represents the three polarisations satisfying $\epsilon_\lambda c^\lambda = 0$. In the rest frame of c, ϵ_λ is, therefore, given by

$$\begin{aligned} \epsilon_\lambda^{(T)} &= (0,1,0,0), \quad (0,0,1,0) \\ \epsilon_\lambda^{(L)} &= (0,0,0,1). \end{aligned} \tag{15}$$

where (T) and (L) signify transverse and longitudinal polarisations, repectively.

In the case where the b is not at rest in the B rest frame ϵ is defined specifically in the B rest frame. Now, if instead of considering the light quark as a spectator, we treat it as an intermediate decay product in an effective theory involving a $\overline{u}bc$ loop, then the relevant traces are

$$M_0^\lambda \propto Tr\left[(\not{c}+m_c)\gamma^\lambda(1-\gamma_5)(\not{b}+m_b)\gamma_5(-\not{u}+m_s)\gamma_5\right] \tag{16}$$

$$M_1^\lambda \propto Tr\left[(\not{c}+m_c)\gamma^\lambda(1-\gamma_5)(\not{b}+m_b)\gamma_5(-\not{u}+m_s)\not{\epsilon}\right] \tag{17}$$

The Suzuki matrix elements are reproduced as $\vec{p}$ goes to zero.

Starting with the B meson at rest,

$$\Gamma(B\to Dl\overline{\nu}) = \frac{1}{2m_B(2\pi)^5}\int \frac{d^3p_D}{2E_D}\frac{d^3p_l}{2E_l}\frac{d^3p_\nu}{2E_\nu}|S|^2\delta^4(B-D-l-\nu) \tag{18}$$

where

$$S = \frac{N^{\frac{1}{2}}G_F V_{cb}V_B V_D}{2\pi}\int \frac{d^3p_b}{2E_u}|\phi^*(p_u)\psi(t_u)|^{\frac{1}{2}}M \tag{19}$$

and where $\vec{p}_u(\vec{p}'_u)$ and $\vec{t}_u(\vec{t}'_u)$ are the up quark momenta in the B and D rest frames, respectively, V_B and V_D are the vertex constants and N is a normalisation. The wavefunctions $\phi(p_u)$ and $\psi(t_u)$ are

$$\phi(p_u) = \frac{1}{\pi^{\frac{3}{2}}p_f^3}exp\left(\frac{-p_u^2}{p_f^2}\right) \quad \text{and} \quad \psi(t_u) = \frac{1}{\pi^{\frac{3}{2}}t_f^3}exp\left(\frac{-t_u^2}{t_f^2}\right) \tag{20}$$

where p_f and t_f are independent adjustable parameters. t_u is given by

$$\begin{aligned} \vec{t}_u^2 &= E_t^2 - m_u^2 \\ &= [(E_uE_D - \vec{p}_u\cdot\vec{p}_D)/m_D]^2 - m_u^2 \end{aligned} \tag{21}$$

where E_t is the energy of the up quark in the D rest frame.

The phase space simplifies as follows:

$$d_3(B\to Dl\overline{\nu}) \propto \tfrac{d^3p_D}{2E_D}\tfrac{d^3p_l}{2E_l}\tfrac{d^3p_\nu}{2E_\nu}\tfrac{d^3p_u}{2E_u}\tfrac{d^3p'_u}{2E'_u}\delta^4(B-D-l-\nu) \tag{22}$$

$$= \tfrac{\pi}{8}\tfrac{dp_Dp_D^2dp_lp_l^2}{E_DE_lE_uE'_u}dcos\theta_l d\phi_l\delta(\nu^2)dp_up_u^2dcos\theta_u d\phi_u dp'_up'^2_u dcos\theta'_u d\phi'_u. \tag{23}$$

$B - D - l - \nu = 0$ and $\nu^2 = 0$ implies

$$(B - D - l)^2 = 0 \tag{24}$$

$$\Rightarrow 2p_D E_l cos\phi_l = 2E_D E_l - 2m_B(E_D + E_l) + (m_D^2 + m_B^2) \tag{25}$$

so the phase space becomes

$$d_s(B \to Dl\overline{\nu}) \propto \frac{\pi^2 p_D}{8E_D} \frac{dp_D dE_l}{E_u E'_u} dp_u p_u^2 dcos\theta_u d\psi_u dp'_u p'^2_u dcos\theta'_u d\psi'_u. \tag{26}$$

If we now insert into this model the parameters given in the previous section we run into problems: it turns out that we only get agreement with ARGUS [16] data for low values of Q^2. This is presumably due to final-state interactions, which in a perturbative QCD framework are expected to grow as one approaches the end-point of the Q^2 distribution. In the exclusive case, QCD corrections are restricted to those which do not create additional hadrons, that is quark propagator self-corrections and vertex corrections. Corrections to the $cD\overline{u}$ vertex are of particular interest because they provide a phenomenological explanation for the discrepancy: the exchange of a gluon between the up and charm quark can reduce their relative momentum, allowing them to combine to form a D or D^* meson.

Conclusion

This model effectively describes the dependence of both inclusive and exclusive semileptonic B decays on the Fermi momentum of the constituent quarks. The parameters that arise naturally in this model agree with those used in other models.

Given that this model describes both inclusive and exclusive decays, we can estimate the rate of semileptonic B decays into D^{**} mesons or clusters consisting of D's or D^* and π's by subtracting the exclusive rates into D and D^* from the inclusive semileptonic B decays into charmed mesons. Experimentally, this rate is found to be between 33% and 41% of the total semileptonic rate[17][18]. This model would appear to have the best chance of accounting for all possible semileptonic decay products of B mesons.

Figure 1: $\frac{dBr}{dE_l}$ for m_u=.13 GeV, m_c=1.4 GeV, m_b=4.9 GeV, p_f=.5 GeV and $V_{ub}/V_{cb} \approx$.07 with data from ARGUS[11][12] and CLEO[13]

Figure 2: $\frac{dBr}{Q^2}$ for m_u=.13 GeV, m_c=1.4 GeV, m_b=4.9 GeV, p_f=.5 GeV and $V_{ub}/V_{cb} \approx$.07 with data from ARGUS[17]

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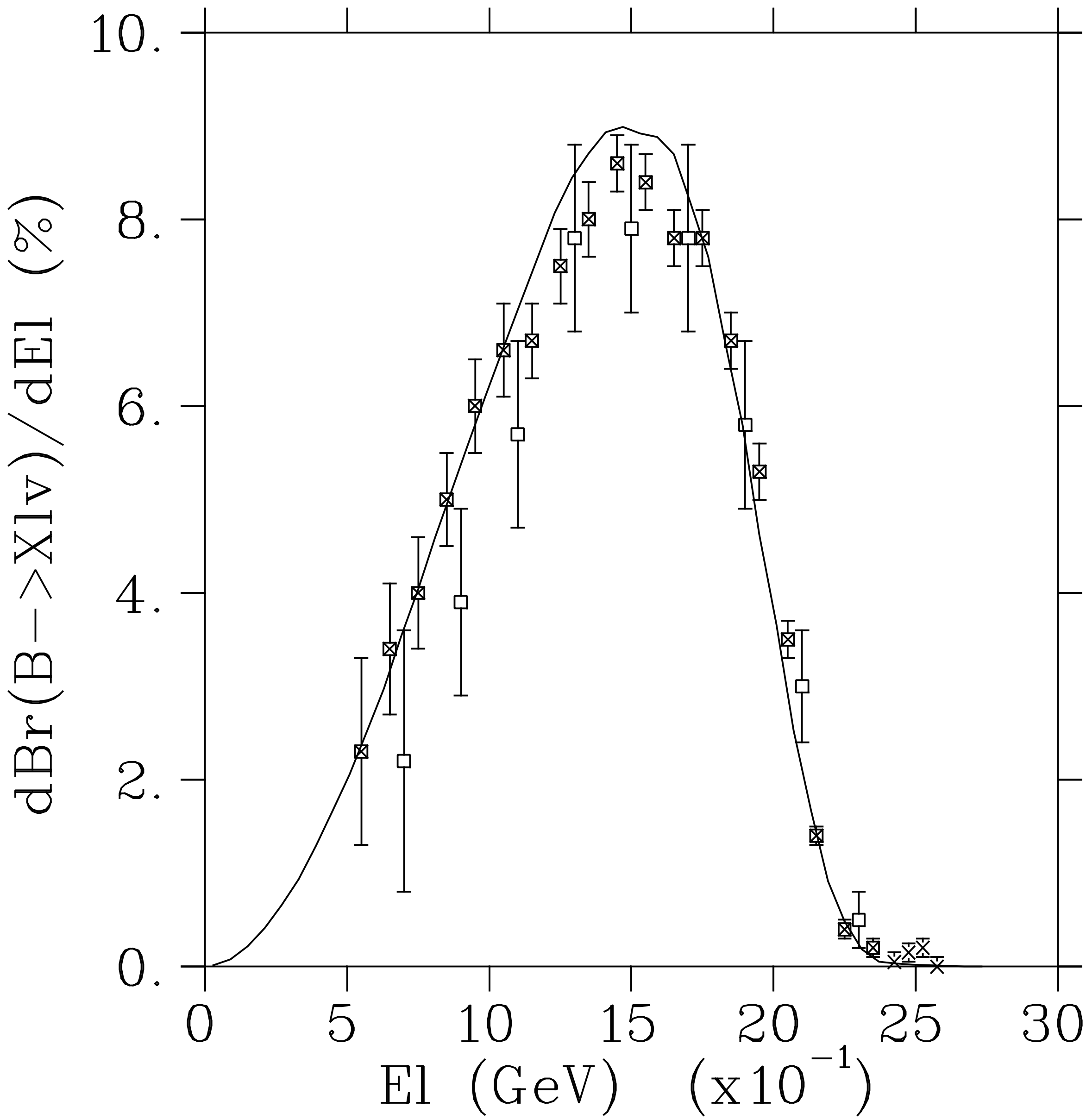

dBr(B−>Xlv)/dEl (%)
El (GeV) (x10^{-1})
0.
2.
4.
6.
8.
10.
0
5
10
15
20
25
30